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A Note on Portfolio Choice For Sovereign Wealth Funds

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Abstract

The vast current account surpluses of commodity-rich nations, combined with record current account deficits in developed markets (US, Britain), have created a new type of investor. Sovereign wealth funds (SWFs) are instrumental in deciding how these surpluses will be invested. We need to better understand their investment problem in order to project future investment flows. Extending Gintschel and Scherer (2004, 2008), we apply the portfolio choice problem for a sovereign wealth fund in a Campbell and Viceira (2002) strategic asset allocation framework. Changing the analysis from a one- to a multi-period framework allows us to establish three-fund separation. We split the optimal portfolio for an SWF into speculative demand as well as hedge demand against oil price shocks and shocks to the short-term risk-free rate. In addition, all terms now depend on the investor's time horizon. We will show that oil-rich countries should hold bonds and that the optimal investment policy for an SWF as a long-term investor is determined by long-term covariance matrices that differ from the correlation inputs that one-period (myopic) investors use.

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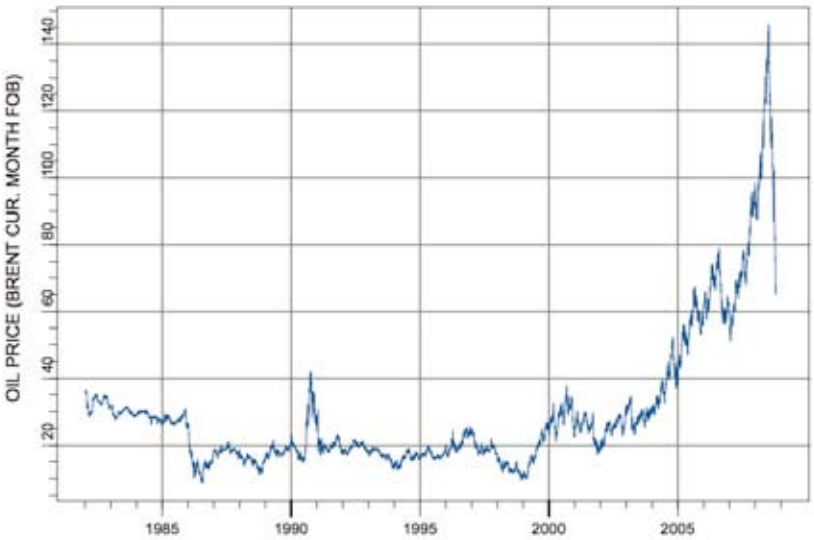
1. Introduction

The vast current account surpluses of commodity-rich nations, combined with record current account deficits in developed markets (US, Britain), have created a new type of investor. Sovereign wealth funds (SWFs) will be instrumental in deciding how these surpluses will be invested. We therefore need to better understand their investment problem.

For many oil-exporting countries, crude oil or gas reserves are the single most important national asset. Any change in the value of reserves directly and materially affects these countries' wealth, and thus the wellbeing of their citizens. Figure 1 serves as an illustration for the violent nature of oil price changes, which can have a destabilizing effect on the economy via volatile real exchange rates.

Figure 1. Daily oil price (Brent current month FOB) movements from January 1982 to September 2008. The underlying total wealth position of an oil-rich country can vary dramatically over time and needs management to smooth intergenerational consumption patterns. Source: Datastream



In recognition of this problem, a number of oil-exporting countries have deposited oil revenues in funds dedicated to financing future expenditures, so-called intergenerational welfare funds. However, oil-based SWFs seem to have been ill prepared for a sudden drop in oil prices. Rather than investing in assets that pay off when oil prices reverse, their assets plunged in parallel. Setser and Ziemba (2009) estimate that the SWFs of Abu Dhabi (ADIA), Kuwait (KIA) and Qatar (QIA) lost 40% of their 2007 value.

Gintschel and Scherer (2004, 2008) identified the optimal asset allocation problem[1] for an SWF as an asset allocation problem with non-tradable wealth that bears a strong resemblance to portfolio choice in the presence of human capital. A country's total wealth can be seen as a combination of financial wealth and non-tradable resource (oil) wealth. According to their one-period framework, assets with negative correlation to oil wealth are well suited to improve the efficiency of total wealth for an oil-rich investor. In their empirical implementation they choose sector portfolios with little or negative correlation to oil price changes. Optimal asset allocation decisions must take these correlations into account in order to avoid welfare losses, *i.e.*, sovereign welfare funds should look for an integrated management of financial and resource wealth.

Doskeland (2007) integrates the SWF into a country's budget; he focuses on social system liabilities, but does not provide closed-form simulations, and he ignores oil price volatility for most of his paper and does not provide a data-generating process capable of modeling mean reversion in both volatility and correlation. No closed-form solutions for portfolio choice are provided.

1 - We will use the term portfolio choice and strategic asset allocation interchangeably. See Köhler and Drobetz (2002) for the importance of strategic asset allocation.

Scherer (2009) introduces resource uncertainty as a form of background risk as well as the impact of optimal oil extraction for an SWF. First, he concludes that uncertainty about the size of a country's oil wealth relative to its total wealth will make it invest less aggressively. In other words, if you don't know how rich you really are you might want to invest less aggressively. Empirically, we should observe that SWFs with larger resource uncertainty should invest less aggressively and vice-versa. We would also expect that economies with low reserves relative to financial wealth are less affected by resource uncertainty. Second, he argues that oil wealth is dependent on the optimal path of extraction policy, which is in turn determined by the interplay between expected future oil prices, extraction costs and the discount factor for future cash flows. He determines an exogenous extraction policy (independent of the evolution of asset returns) and investigates the impact on portfolio choice on the sequence of one-period optimization problems. As long as financial wealth is low relative to resource wealth, an SWF will need to invest aggressively to have a meaningful impact on total wealth. Depending on the speed of the optimal extraction policy, a maturing SWF will gradually invest less aggressively with less appetite for aggressive hedging or speculative risk taking.

We will extend the previous work in two directions. First, we add a time dimension, which is missing from the one-period framework of other work. Introducing a multi-period setting (*i.e.*, an autoregressive data-generating process) with return predictability allows us to investigate how asset allocation recommendations change as the time horizon becomes larger. This is of particular relevance for SWFs as long-term investors. Campbell and Viceira (2002), for example, have shown that stocks are less risky in the long run. These "term structure effects" created by the data-generating process might also have an effect on the term structure of the correlation of various asset classes and oil. In other words, we investigate the term structure of hedging demand. What happens if the correlation of bonds and oil increases over time or if the volatility of bonds decreases relative to the volatility of oil wealth? In both cases we would expect the optimal hedging demand to increase. Here a (short-) long-term investor will use a (short-) long-term measure of correlation. Second, we apply our framework to broad economic asset classes like equities, bonds and listed real estate[2] in order to give observers of SWFs some ideas where global funds are flowing. If SWFs have a natural demand for government bonds as the ultimate recession hedge, this is good news for countries like the US that require a constant inflow of capital to finance their current account deficit and to stabilize the US dollar.

The outline is as follows. Section 2 applies the portfolio choice problem for an oil-based SWF in a multi-period setting using the framework of Campbell and Viceira (2002), *i.e.*, we introduce time into the myopic asset allocation framework discussed so far in the literature. After establishing three-fund portfolio separation, we describe our data and data-generating process in section 3. Section 4 presents the term structure of optimal hedging demand, *i.e.*, the implications of asset price dynamics for the asset allocation problem of an SWF at different time horizons. Section 5 concludes.

2. Three-Fund Separation - Incorporating the SWF into Government Budgets

As in Gintschel and Scherer (2008), we view the optimal asset allocation problem of an SWF as the decision-making problem for an investor with non-tradable endowed wealth (oil reserves). To transform their solution into the multi-period framework of Campbell and Viceira (2002) we let x_t denote our vector of asset excess returns (including oil) over cash as well as cash and s_t the vector of economic state variables, *i.e.*, conditioning information. The idea here is that only unexpected variations (those not explained by state variables) pose a risk. We denote the (conditional) annualized n period return and risk by

$$(1) \qquad \mu^{(n)} = n^{-1} E\left[\sum_{i=1}^{n} x_{t+n} \middle| s_t \right]$$

2 - We will focus on portfolio investments. This is a narrower brief than what sovereign investors can do. Rather than investing in securities (mostly USD-dominated) abroad, sovereign investors can also use their oil revenues to build exposure to future growth industries and develop the necessary infrastructure to make their country an attractive place to attract top human talent. Dubai and Qatar are prime examples. During the following exposition we will rely on the normality in return distributions assumption to allow us to come up with closed-form solutions that provide conceptual insight into the structure of the underlying problem. While we are aware that returns on capital markets and certainly on commodities like oil are in the short run far from normal, we also believe an SWF belongs to the group of long-term investors, such that the central limit theorem will help mitigate the non-normality issue.

$$(2) \qquad \Sigma^{(n)} = n^{-1}\mathrm{Var}\left(\sum_{i=1}^{n} x_{t+n} \middle| s_t\right)$$

where $\mu^{(n)}$ and $\Sigma^{(n)}$ represent *n* period expected returns. We can also define sub-vectors and sub-matrices. To extract the required *n*-period risk and return measures we use $\mu_a^{(n)}$ for the expected excess returns for assets, $\Sigma_{aa}^{(n)}$ for the covariance matrix of asset returns, $\Sigma_{ao}^{(n)}$ for the covariance vector of asset and oil returns and $\Sigma_{ac}^{(n)}$ for the covariance of asset excess returns and short-term cash rates. Equations (1) and (2) describe the term structure of risk and return. This information is used to specify the mean-reversion properties of various asset classes to introduce a time dimension to portfolio choice. If, for example, we found considerable mean reversion in equity returns, long-term investors could invest more aggressively in equities than short-term investors. Suppose a sovereign entity holds both financial assets in an SWF and oil assets underground, and let $0 < \theta \leq 1$ denote the fraction of financial wealth to total wealth (oil plus financial assets).[3] Given CRRA preferences with risk-aversion parameter γ, we show that the optimal portfolio for horizon *n* is given by[4]

$$(3) \qquad w_t^{(n)} = \frac{1}{\gamma}\left(\frac{1}{\gamma}\Sigma_{aa} + \frac{(\gamma-1)}{\gamma}\Sigma_{aa}^{(n)}\right)^{-1}\left[\frac{1}{\theta}\left(\mu_a^{(n)} + \frac{1}{2}\sigma_a^2\right) - (\gamma-1)\frac{(1-\theta)}{\theta}\Sigma_{ao}^{(n)} - \frac{1}{\theta}(\gamma-1)\Sigma_{ac}^{(n)}\right]$$

This is equivalent to three-fund separation.[5] The SWF wants to hold a leveraged ($\frac{1}{\theta} > 1$) speculative portfolio that depends purely on expected asset excess returns, $w_{spec}^{(n)}$, an equally (as changes in the risk-free rate can only be managed via the asset portfolio, even though they affect total wealth) leveraged hedging portfolio, $w_{hedge,cash}^{(n)}$, that protects against inter-temporal variations in short-term rates and another hedging portfolio that protects against oil price shocks, $w_{hedge,oil}^{(n)}$.[6]

$$(4) \qquad w_{spec}^{(n)} = \left(\frac{1}{\theta}\right)\left(\frac{1}{\gamma}\right)\left(\frac{1}{\gamma}\Sigma_{aa} + \frac{\gamma-1}{\gamma}\Sigma_{aa}^{(n)}\right)^{-1}\left(\mu^{(n)} + \frac{1}{2}\sigma_a^2\right)$$

$$(5) \qquad w_{hedge,cash}^{(n)} = -\left(\frac{1}{\theta}\right)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{1}{\gamma}\Sigma_{aa} + \frac{\gamma-1}{\gamma}\Sigma_{aa}^{(n)}\right)^{-1}\Sigma_{ac}^{(n)}$$

$$(6) \qquad w_{hedge,oil}^{(n)} = -\left(\frac{1-\theta}{\theta}\right)\left(\frac{\gamma-1}{\gamma}\right)\left(\frac{1}{\gamma}\Sigma_{aa} + \frac{\gamma-1}{\gamma}\Sigma_{aa}^{(n)}\right)^{-1}\Sigma_{ao}^{(n)}$$

For $\theta = 1$ we arrive at the optimal solution for an investor with financial wealth only, while for $\gamma \to \infty$ we get $w_{hedge,cash}^{(n)} = -\left(\frac{1}{\theta}\right)\left[\Sigma_{aa}^{n}\right]^{-1}\Sigma_{ac}^{(n)}$ and $w_{hedge,oil}^{(n)} = -\left(\frac{1-\theta}{\theta}\right)\left[\Sigma_{aa}^{(n)}\right]^{-1}\Sigma_{ao}^{(n)}$. For $\gamma = 1$, *i.e.*, a log investor, we arrive at the familiar solution that there is no hedging demand and the optimal solution degenerates to the one-period myopic speculative demand $w_{spec}^{(n)} = \frac{1}{\theta}\Sigma_{aa}^{-1}\left(\mu^{(n)} + \frac{1}{2}\sigma_a^2\right)$. This framework is extremely general and its application is not limited to equities or bonds, but can easily be applied to hedge funds, private equity investments, etc.[7]

3. Return Dynamics and the Data-Generating Process

3.1 Model and Data

Our model is incomplete without a specification for (1) and (2). We start by assuming a first-order vector auto-regression as our data-generating process (DGP):

$$(7) \qquad w_{spec}^{(n)} = \frac{1}{\theta}\Sigma_{aa}^{-1}\left(\mu^{(n)} + \frac{1}{2}\sigma_a^2\right)$$

3 - We assume θ to be exogenous (known). This assumption is necessary to allow tractability. An endogenous θ would require us to simultaneously determine asset mix and optimal extraction policy.

4 - See appendix A for a derivation of this result.

5 - As times goes by, the investor would change her asset allocation from $w^{(n)}$ to $w^{(n-1)}$ and to $w^{(n-2)}$ etc.

6 - The optimal position in risk-free assets is given by $1 - \iota^T w^{(n)}$.

7 - Including investments with less data availability can be dealt with by applying standard econometric techniques to deal with time series of different lengths. Suppose we add a vector of new time series with shorter history, y_t. Following Stambaugh (1997), all that is needed is to "glue" the new time series to the existing VAR in the form $y_{t+1} = c + Dz_{t+1} + Ez_t + Fy_t + \eta_{t+1}$, where $\eta_{t+1} \sim N(0, \Sigma)$ and *F* is a diagonal matrix, *i.e.*, the new series is allowed to exhibit autocorrelation, but it does not help forecast other series or state variables. The contemporaneous covariance is measured by *D* and as such we assume ε_{t+1} and η_{t+1} are uncorrelated (all common variation is caught in *D*). Including the new series leads us to the new VAR representation $z_{t+1}^* = \Im_0 + \Im_1 z_t^* + e_{t+1}$ where

$$z_{t+1}^* = \begin{bmatrix} z_{t+1} \\ y_{t+1} \end{bmatrix}, \Im_0 = \begin{bmatrix} a \\ c + Da \end{bmatrix}, \Im_1 = \begin{bmatrix} B & 0 \\ E + DB & F \end{bmatrix}, \Omega^* = \begin{bmatrix} \Omega & \Omega D^T \\ D\Omega & \Sigma + D\Omega D^T \end{bmatrix}$$

We can test the restrictions implicit in $\Im_0, \Im_1$ on an overlapping data set.

where $z_t = [x_t \quad s_t]^T$, Ω represents the residual covariance matrix of our VAR with a vector of constants, *a*, and a coefficient matrix *B*. At this stage the reader might worry about the validity of (7). The residuals for the oil equation, for example, are most likely not normally distributed. However, this would still leave the coefficient estimates unbiased. In addition, we could simulate (7) as a semi-parametric VAR, *i.e.*, by bootstrapping from the empirical residuals instead of Ω. However, over a larger number of periods the central limit theorem will provide approximately normal variables. Finally, we feel that (7) is widely used in inter-temporal portfolio choice problems. Of course, one could think of extensions.[8] For example, we could estimate a co-integrating VAR in levels rather than a stationary VAR in first differences to capture the speeds of mean reversion and different equilibrium relations), but we feel this is beyond the scope of this paper, namely to lay out a framework. From (7) we can easily work out the multi-period expressions for risk and return in (1) and (2) as[9]

$$\Sigma^{(n)} = \sum_{i=1}^{n}\left[\left(\sum_{j=0}^{i-1} B^j\right)\Omega\left(\sum_{j=0}^{i-1} B^j\right)^T\right] \tag{8}$$

where again $\Sigma^{(n)}$ is the covariance matrix of *n* period returns and $B^0 = I$. Correlation, variance and covariance can be easily picked from the elements in $\Sigma^{(n)}$. Plotting variances, correlations, etc. against *n* provides a term structure of risk.

We proxy the investment universe for an SWF contained in x_t by the CRSP Value weighted stock index, real estate investments (FTSE NAREIT real estate index), government bonds (Lehman Long US Treasury Bonds Index), one-month Treasury bill rate and real bonds (TIPS).[11] For changes in oil wealth, we use Brent current month FOB. Given very high correlations between Brent, West Texas Intermediate and Arab Light the choice of variable does not matter for our purposes. Economic state variables, s_t, are given by dividend yield, credit spread (the difference between Baa corporate bond yield and the ten-year Treasury constant maturity rate), term spread (difference between the ten-year Treasury constant maturity rate and one-month T-Bill rate) and nominal yield (ten-year Treasury constant maturity rate).

Table 1. Summary Statistics. We report descriptive statistics for all endogenous (assets and state variables) variables in our VAR. Only the Sharpe ratios have been annualized.

	Symbol	Mean (μ)	Volatility (σ)	Sharpe ($\frac{\mu - rc}{\sigma}\sqrt{4}$)	Min	Max	Skew	Kurtosis
Assets								
Oil	ro_t	-0.37	19.23	-0.04	-0.89	0.82	-0.27	6.33
Tips	rr_t	0.42	0.60	1.40	-0.02	0.02	-0.94	2.29
T-Bill	rc_t	1.29	0.59		0.00	0.03	0.32	0.12
Long Bonds	rb_t	1.33	4.97	0.54	-0.09	0.17	0.51	0.37
Equities	re_t	1.69	8.14	0.42	-0.28	0.18	-0.79	1.46
Real Estate	ri_t	1.42	7.03	0.40	-0.17	0.19	0.00	0.05
State Variables								
Nominal	yn_t	7.15	2.56		0.04	0.15	0.96	0.36
Credit Spread	cs_t	1.96	0.50		0.01	0.04	1.04	0.67
Time Spread	ts_t	1.75	1.25		-0.01	0.05	-0.05	-0.62
Dividend Yield	dy_t	-3.66	34.24		-4.55	-2.97	-0.21	-0.58

8 - See Campbell, Chan and Viceira (2003), Campbell *et al.* (2004), Guidolin and Timmerman (2007), to name a few.
9 - See appendix B for a brief sketch.
10 - An alternative (brute force) way to calculate the covariance of multi-period returns is to simulate $z_{t+1} + z_{t+2} + \ldots + z_{t+n}$ from (7) and directly calculate the required statistics.
11 - TIPS are constructed using the methodology in Kothari and Shanken (2004).

Our analysis is based on quarterly returns from 1973 I through 2007 IV. Table 1 summarizes the data and our notation. Oil returns are statistically not different from cash rates, *i.e.*, excess returns are not statistically different from zero. This supports the Hotelling-Solow rule under perfectly integrated capital markets. Natural resource prices should grow at world interest rates such that countries are indifferent between depletion (earning the interest rate) and keeping oil underground (earning price changes). Our state variables are common in the academic literature and motivated by time variation in investment opportunities[12] and the investment universe will span most of the investment opportunity set for an SWF.

3.2 Return Dynamics

We start with calibrating our DGP in (7) to the data in table 1. Coefficient estimates and R^2 are given in table 3 while table 2 provides estimates for Ω with the main diagonal representing quarterly volatility. Unexplained quarterly real estate volatility amounts to 6.65%, which is only marginally smaller than unconditional volatility of 7.03%. This is hardly surprising, given that the R^2 for the real estate equation is extremely low. Dividend yields have significant forecasting power for equities and bonds, while bonds can also be forecasted using the last quarters' nominal yields. Shocks on the dividend yields show negative contemporaneous correlation (-0.27) with equity returns.

Table 2. Results from first-order VAR: Residual covariance matrix. We report the error covariance matrix, Ω, of the VAR from (7). The main diagonal contains (quarterly) volatility. while off-diagonal entries represent correlations.

	rr_t	rc_t	rb_t	re_t	ro_t	ri_t	yn_t	cs_t	ts_t	dy_t
rr_t	**0.47**									
rc_t	0.05	**0.12**								
rb_t	-0.45	-0.08	**4.28**							
re_t	-0.28	-0.07	0.10	**7.11**						
ro_t	0.54	0.09	-0.22	-0.31	**17.84**					
ri_t	-0.40	-0.16	0.26	0.55	-0.28	**6.65**				
yn_t	0.20	0.08	-0.28	-0.15	-0.08	-0.17	**0.25**			
cs_t	-0.01	-0.19	0.02	-0.01	-0.01	0.01	-0.54	**0.18**		
ts_t	0.09	-0.33	-0.24	0.11	-0.21	0.02	0.47	-0.19	**0.43**	
dy_t	0.05	-0.08	0.04	-0.27	-0.02	-0.31	0.18	-0.21	0.03	**13.85**

12 - See Campbell and Viceira (2002).

Table 3. Results from first-order VAR: Parameter Estimates. We report the coefficients of the VAR in equation (7) together with standard errors and *t*-values. See Table 1 for a description of individual variable names. The last two rows contain the "R-squared" as well as "adjusted R-squared" for each individual regression. Coefficients significant at the 5% level are in bold.

	z_t									
	x_t						s_t			
	rr_t	rc_t	rb_t	re_t	ro_t	ri_t	yn_t	cs_t	ts_t	dy_t
a	**0.04**	-0.00	**-0.41**	**0.57**	0.62	-0.02	**0.01**	**-0.01**	**0.05**	**-1.30**
	0.01	0.00	**0.11**	**0.18**	0.46	0.17	**0.01**	**0.00**	**0.01**	**0.36**
	3.23	-0.61	**-3.73**	**3.08**	1.34	-0.12	**2.13**	**-2.02**	**4.76**	**-3.64**
rr_{t-1}	0.08	**-0.06**	**2.28**	**-4.29**	-5.77	-1.13	0.06	-0.02	0.14	-1.61
	0.13	**0.03**	**1.14**	**1.90**	4.76	1.77	0.07	0.05	0.12	3.70
	0.65	**-1.97**	**2.00**	**-2.26**	-1.21	-0.64	0.85	-0.37	1.24	-0.44
rc_{t-1}	-0.22	1.45	**-8.42**	-5.27	-7.36	**-12.23**	0.65	-0.22	**-5.65**	15.38
	0.44	0.11	**3.97**	6.59	16.53	**6.16**	0.24	0.17	**0.40**	12.83
	-0.51	13.51	**-2.12**	-0.80	-0.45	**-1.99**	2.76	-1.36	**-14.04**	1.20
rb_{t-1}	-0.02	**-0.01**	0.10	**0.42**	**-1.25**	0.13	**-0.10**	**0.03**	**-0.03**	-0.32
	0.01	**0.00**	0.11	**0.18**	**0.44**	0.16	**0.01**	**0.00**	**0.01**	0.34
	-1.51	**-5.14**	0.95	**2.43**	**-2.86**	0.78	**-16.22**	**7.52**	**-2.88**	-0.94
re_{t-1}	-0.01	0.00	0.12	0.05	-0.12	0.04	0.00	**-0.01**	0.00	-0.05
	0.01	0.00	0.07	0.11	0.29	0.11	0.00	**0.00**	0.01	0.22
	-1.71	0.10	1.75	0.45	-0.41	0.36	0.28	**-3.41**	0.71	-0.24
ro_{t-1}	-0.00	0.00	0.02	**0.12**	-0.09	0.04	0.00	-0.00	0.00	0.09
	0.00	0.00	0.03	**0.05**	0.12	0.05	0.00	0.00	0.00	0.09
	-0.71	0.75	0.52	**2.55**	-0.75	0.98	0.12	-0.19	0.54	1.01
ri_{t-1}	0.00	0.00	-0.16	-0.24	0.08	-0.01	0.01	**-0.01**	-0.01	0.04
	0.01	0.00	0.09	0.15	0.37	0.14	0.01	**0.00**	0.01	0.28
	0.50	0.67	-1.84	-1.63	0.23	-0.11	1.03	**-2.16**	-0.88	0.13
yn_{t-1}	-0.11	**-0.11**	**3.33**	-0.26	-0.96	2.71	**0.80**	0.08	**1.17**	-1.12
	0.11	**0.03**	**0.95**	1.58	3.96	1.48	**0.06**	0.04	**0.10**	3.07
	-1.05	**-4.33**	**3.51**	-0.17	-0.24	1.83	**14.13**	1.92	**12.16**	-0.37
cs_{t-1}	-0.02	**-0.07**	-1.12	-0.04	1.53	1.42	0.09	**0.82**	0.12	0.31
	0.11	**0.03**	0.99	1.65	4.13	1.54	0.06	**0.04**	0.10	3.21
	-0.18	**-2.52**	-1.13	-0.02	0.37	0.92	1.61	**19.89**	1.17	0.10
ts_{t-1}	0.06	**0.18**	-1.17	-0.80	-1.97	-1.91	**0.14**	-0.04	**-0.60**	6.74
	0.10	**0.02**	0.90	1.50	3.77	1.40	**0.05**	0.04	**0.09**	2.92
	0.63	**7.33**	-1.29	-0.53	-0.52	-1.36	**2.53**	-1.11	**-6.59**	2.31
dy_{t-1}	**0.01**	-0.00	**-0.09**	**0.12**	0.11	-0.00	**0.00**	**-0.00**	**0.01**	**0.71**
	0.00	0.00	**0.02**	**0.04**	0.10	0.04	**0.00**	**0.00**	**0.00**	**0.08**
	2.51	-0.98	**-3.63**	**2.86**	1.09	-0.07	**2.28**	**-2.88**	**4.24**	**8.86**
R^2	0.35	0.96	0.26	0.22	0.14	0.11	0.99	0.88	0.88	0.83
R^2	0.28	0.96	0.18	0.14	0.04	0.01	0.99	0.86	0.86	0.81

At the same time, rising dividend yields impact future equity returns positively (positive significant regression coefficient of 0.12). In summary: positive shocks to dividend yields (an increase) negatively impact current returns but positively impact next period returns. This creates mean reversion and is likely to reduce long-term risk. All state variables are very persistent, as the high R^2 and the large highly significant autoregressive coefficients indicate. Also note that the system (7) is stable, as all Eigen-values of B have a modulus less than 1 (the largest is 0.94). Hence, the process is stable and therefore also stationary. The results also confirm recent work by Driesprong *et al.* (2008), who found that lagged oil price changes predict future equity market returns. We find a t-value of 2.55 on quarterly data, which is higher than their t-values using weekly data.[13]

13 - However, our regression coefficient shows the opposite sign, but since their regression left many statistically significant explanatory variables out, we would not be surprised if their results were biased.

4. The Term Structure of Hedging Demand

In this section we will focus exclusively on the hedging demand for an SWF as $w_{t,spec}^{(n)}$ is essentially the same for an asset-only investor, an asset-liability investor or an asset-only investor with non-tradable wealth and hence not specific to an SWF.[14] We ignore effects from γ and θ as they only affect leverage. The results are summarized in figure 2 and figure 3. In both cases, hedging demand is positive as long as correlation of investment and risk source is negative. In other words: assets that show negative correlation with oil or time-varying short rates will get positive weights. While correlation drives the sign of hedging demand, it is the relative term structure of risk (does annualized volatility grow or decay with time horizon?) that determines, all else being equal, the extent of hedging demand, which essentially equals a "beta" estimate. For example, $\left[\Sigma_{aa}^{(n)}\right]^{-1}\Sigma_{ao}^{(n)}$ describes a vector of asset betas relative to oil.

Figure 2. Hedging time variation in short-term interest rates. We represent the hedge portfolio according to $w_{hedge,cash}^{(n)} = -\frac{1}{\theta}\left[\Sigma_{aa}^{n}\right]^{-1}\Sigma_{ac}^{(n)}$ for $n = 1,...,30$ and $\theta = \frac{1}{2}$.

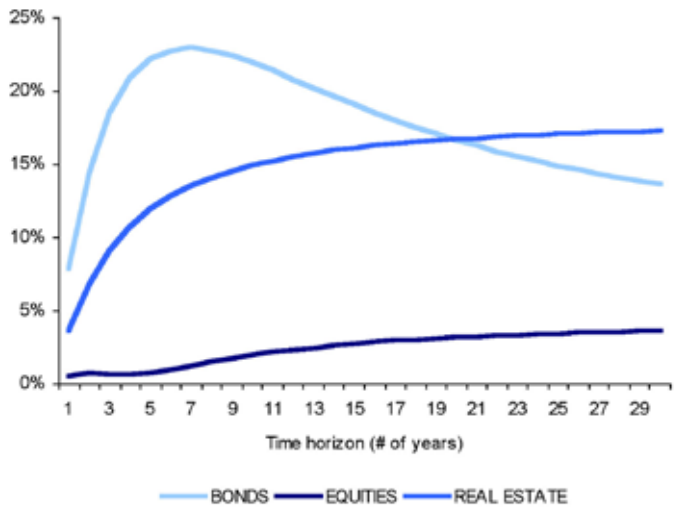
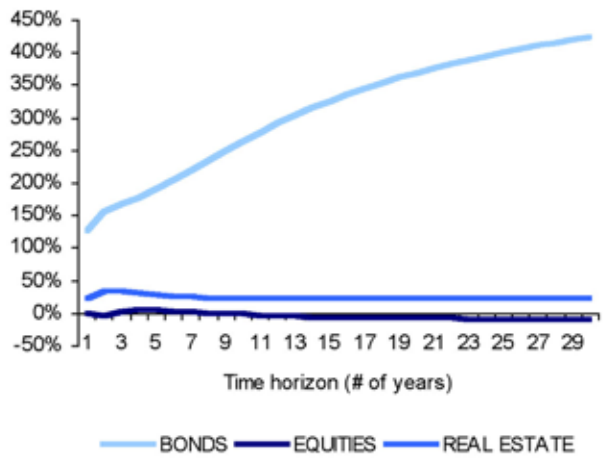


Figure 3. Hedging oil price shocks over time. We represent the hedge portfolio according to $w_{hedge,oil}^{(n)} = -\left(\frac{1-\theta}{\theta}\right)\left[\Sigma_{aa}^{(n)}\right]^{-1}\Sigma_{ao}^{(n)}$ for $n = 1,...,30$ and $\theta = \frac{1}{2}$.



Long government bonds play the biggest role in both hedge portfolios, as government bonds are both a recession hedge (they increase in value when oil prices come down in a recession) and a hedge against deteriorating investment opportunities, *i.e.*, falling rates. While any empirical result is subject to estimation error,[15] we observe that the sign of our hedging demand coincides with our economic prior. Government bonds are a recession hedge and tend to pay off in those states of the world where demand for commodities is low and prices are depressed. While our data set ends at the end of 2007 we observe that this hedge would have worked extremely well in 2008, where all asset classes experienced record losses and oil fell from USD150 to less than USD40 a barrel. Only government bonds performed well, with the ten-year Treasury note up about 20%.

14 - We are therefore not required to create forecasts for the asset classes involved. This topic is well covered in Drobetz (2005).
15 - See Fabozzi *et al.* (2008) for an application of Gibbs sampling on models like (7).

5. Conclusions

We have extended the portfolio choice problem for oil-rich sovereign investors from a one-period to a multi-period model. This results in separate terms for speculative demand as well as hedging demand for inter-temporal variations in short rates and shocks to oil wealth. We arrive at three-fund separation. In addition, all terms depend now on the investor's time horizon. This contrasts with earlier work that neglected the impact of mean reversion on portfolio choice for an SWF. We also provided an empirical illustration for our framework including equities, bonds and listed real estate. Our analysis concludes that an SWF should hold a considerable amount of assets in long US government bonds in order to hedge both the negative effects of oil price shocks and deteriorating short rates. Empirically, we find confirmation in the fact that part of the growing current account surplus of commodity exporters has been invested in US Treasury bonds. However, with SWF funds recording losses of about 40% in 2008 while government bonds yielded about 20% in the same period, they did not allocate nearly enough.

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Appendix A

We can write the n period return, $\mu_{t,SWF}^{(n)}$, and risk, $\sigma_{t,SWF}^{(n)2}$, as a function of our decision vector of portfolio weights, $w_t^{(n)}$, according to

$$(1) \quad \mu_{t,SWF}^{(n)} = \theta w_t^{(n)T}\left(\mu_a^{(n)} + \tfrac{1}{2}\sigma_a^2\right) - \tfrac{1}{2}\theta w_t^{(n)T}\Sigma_{aa}\theta w_t^{(n)}$$

$$(2) \quad \sigma_{t,SWF}^{(n)2} = \theta^2 w_t^{(n)T}\Sigma_{aa}^{(n)} w_t + 2\theta(1-\theta) w_t^{(n)T}\Sigma_{ao}^{(n)} + 2\theta w_t^{(n)T}\Sigma_{ac}^{(n)}$$

In accordance with the literature, we assume $w_t^{(n)}$ to remain constant, which equates to assuming an investor with constant time horizon n. Assuming CRRA, we maximize $\frac{1}{1-\gamma}\left(W_{SWF,t+n}\right)^{1-\gamma}$ and the optimization problem becomes

$$(3) \quad \max_{w_t^{(n)}}\left[\mu_{t,SWF}^{(n)} + \tfrac{1}{2}(1-\gamma)\sigma_{t,SWF}^{(n)2}\right].$$

Differentiating (3), we get the first-order conditions

$$(4) \quad \theta\left(\mu_a^{(n)} + \tfrac{1}{2}\sigma_a^2\right) - \theta^2\Sigma_{aa} w_t^{(n)} + \tfrac{(1-\gamma)}{2}\theta^2 2\Sigma_{aa}^{(n)} w_t^{(n)} + \tfrac{(1-\gamma)}{2}2\theta(1-\theta)\Sigma_{ao}^{(n)} + \tfrac{(1-\gamma)}{2}2\theta\Sigma_{ac}^{(n)} = 0$$

We divide by θ, collect all terms involving $w_t^{(n)}$ on the left side and use the identity that $(1-\gamma) = -(\gamma-1)$ to finally arrive at

$$(5) \quad w_t^{(n)} = \tfrac{1}{\gamma}\left(\tfrac{1}{\gamma}\Sigma_{aa} + \tfrac{(\gamma-1)}{\gamma}\Sigma_{aa}^{(n)}\right)^{-1}\left[\tfrac{1}{\theta}\left(\mu_a^{(n)} + \tfrac{1}{2}\sigma_a^2\right) - (\gamma-1)\tfrac{(1-\theta)}{\theta}\Sigma_{ao}^{(n)} - \tfrac{1}{\theta}(\gamma-1)\Sigma_{ac}^{(n)}\right]$$

which is identical to equation (3) in the main text.

Appendix B

We want to shed more light on equation (8) in the main text. For illustrative purposes we first present a two-period example which we generalize afterwards. The two-period returns from a first-order VAR are given by

$$(1) \quad \begin{aligned} z_{t+1} + z_{t+2} &= a + Bz_t + \varepsilon_{t+1} + a + Bz_{t+1} + \varepsilon_{t+2} \\ &= a + Bz_t + \varepsilon_{t+1} + a + B\left(a + Bz_t + \varepsilon_{t+1}\right) + \varepsilon_{t+2} \\ &= a + Bz_t + \varepsilon_{t+1} + a + BA + B^2 z_t + B\varepsilon_{t+1} + \varepsilon_{t+2} \end{aligned}$$

However, uncertainty arises only from the residual terms, as the model parameters are assumed to be fully known in t (no estimation error) as well as z_t. We then get

$$(2) \quad \begin{aligned} \Sigma^{(2)} &= Var\left(z_{t+1} + z_{t+2}\right) = Var\left(\varepsilon_{t+1} + B\varepsilon_{t+1} + \varepsilon_{t+2}\right) \\ &= Var\left(\varepsilon_{t+1}(I+B) + \varepsilon_{t+2}\right) \\ &= (I+B)\Omega(I+B)^T + \Omega \end{aligned}$$

The n period generalization follows the same logic as (2) and can be calculated via the following expression

(3)

$$\begin{aligned} \Sigma^{(n)} &= \Omega \\ &+ (I+B)\Omega(I+B)^T \\ &+ \left(I+B+B^2\right)\Omega\left(I+B+B^2\right)^T \\ &+ \ldots \\ &+ \left(I+B+B^2+\ldots+B^{n-1}\right)\Omega\left(I+B+B^2+\ldots+B^{n-1}\right)^T \\ &= \sum_{i=1}^{n}\left[\left(\sum_{j=0}^{i-1}B^j\right)\Omega\left(\sum_{j=0}^{i-1}B^j\right)^T\right] \end{aligned}$$

which is exactly what we find in the main text.